

# Tomographic reconstruction for three-photon imaging

Art & Science - INPACT

13/02/2024

---

Mehdi Latif<sup>1,2</sup>, Jérôme Idier<sup>1</sup>, Thomas Carlier<sup>2</sup>, Simon Stute<sup>2</sup>

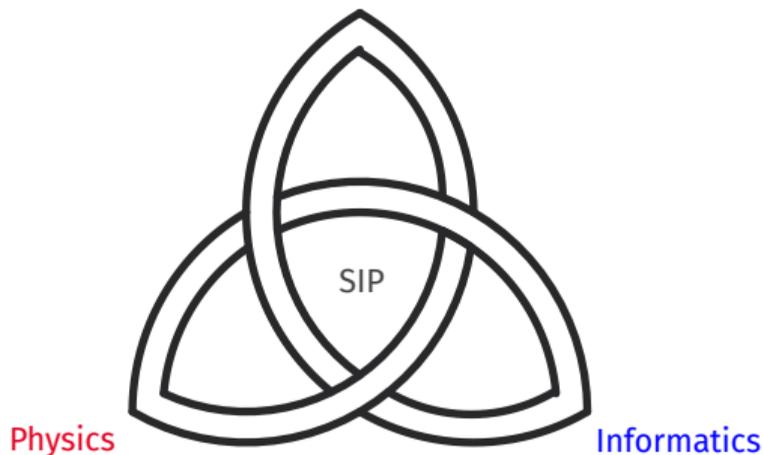
<sup>1</sup>SiMS,LS2N - Ecole Centrale Nantes,

<sup>2</sup>Nuclear Oncology, CRCI<sup>2</sup>NA - CHU de Nantes



Tomographic reconstruction for three-photon ( $3\gamma$ ) imaging

Mathematics



Inspired by P. Flandrin, shared by B. Pascal

What's SIP ?

**Physics:** to describe the observed phenomena;

**Mathematics:** to model the observed phenomena;

**Informatics:** to implement efficient algorithms & simulate observed phenomena.



# Physics

*To describe the observed phenomena*

# The origin: XENon Medical Imaging System (XEMIS)

**Project start-up:** 2004 by XENON team - Subatech laboratory (Nantes, France).

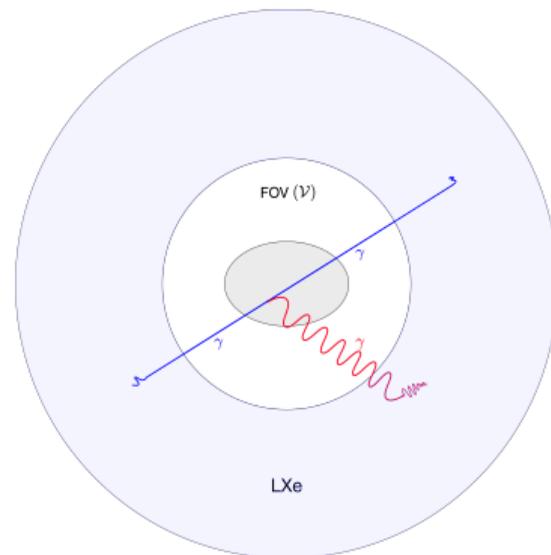
XEMIS - Grignon et al. [2007]

*Low-activity medical imaging*

Development of the **Compton camera** with **liquid xenon (LXe)**;

Based on the **three-photon ( $3\gamma$ )** imaging technique

↪ A camera **like no other** !!



$3\gamma$  imaging technique in XEMIS LXe continuous detector.

# The origin: XENon Medical Imaging System (XEMIS)

**Project start-up:** 2004 by XENON team - Subatech laboratory (Nantes, France).

XEMIS - Grignon et al. [2007]

*Low-activity medical imaging*

Development of the **Compton camera** with **liquid xenon (LXe)**;

Based on the **three-photon ( $3\gamma$ )** imaging technique

↪ A camera **like no other !!**

XEMIS2 - Gallego Manzano [2016]

LXe **cylindrical & pre-clinical camera**  
for rodents **total-body** imaging

Up to now ...

Efforts are spent in **experimental design** of the camera.

<sup>1</sup>Source: Subatech laboratory.



XEMIS2 sketch<sup>1</sup>



XEMIS2 camera - CHU-CIMA Nantes<sup>1</sup>

# three-photon ( $3\gamma$ ) imaging - 1/2

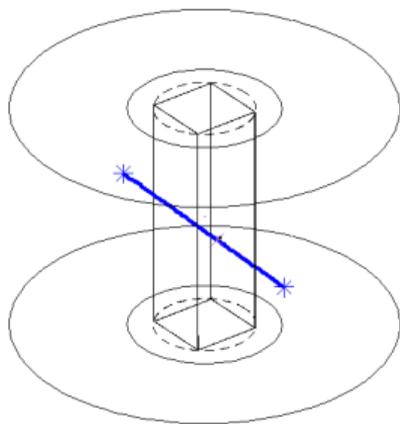
$3\gamma$  imaging: a Functional imaging method

Injection of a radiotracer + isotope  $\rightsquigarrow$  an **image** of the **radioactive distribution** in tissues.

Based on the combination of **two existing** imaging techniques & a **specific** isotope:

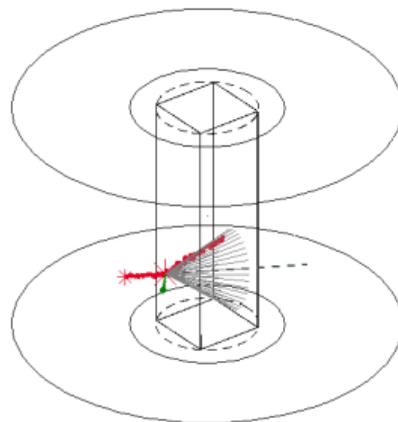


with  $E_0 = 1.157\text{MeV}$ .



PET imaging  $\rightsquigarrow M^{\text{PR}} \in \text{LOR}$

$$\mathcal{L} := (V_1, V_2)$$



Compton imaging  $\rightsquigarrow M \in \text{COR}$

$$\mathcal{C} := (V_1', \overrightarrow{V_2'V_1'}, \beta) \text{ \& } \beta = f(E_0, E_1)$$

## three-photon ( $3\gamma$ ) imaging - 2/2

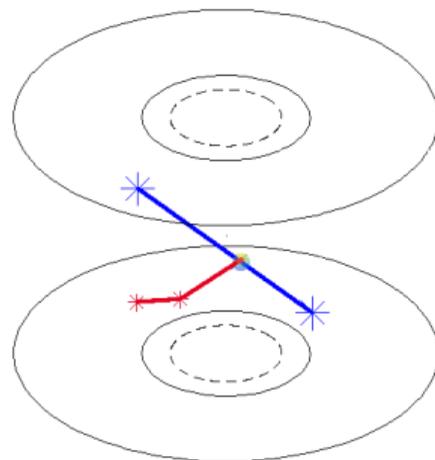
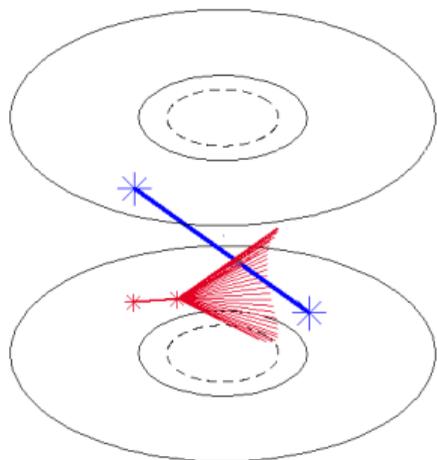
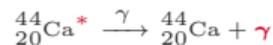
$3\gamma$  imaging: a Functional imaging method

Injection of a radiotracer + isotope  $\rightsquigarrow$  an **image** of the **radioactive distribution** in tissues.

Based on the combination of **two existing** imaging techniques & a **specific** isotope:



with  $E_0 = 1.157\text{MeV}$ .



$\rightsquigarrow$  The source of the radioactive decay is **close** to the **LOR/COR** intersection.

# Classes of detections

XEMIS2 has been developed to detect  $3\gamma$  emissions - Gallego Manzano [2016].

**But** there's more ...

## Classes of detection:

### Complete detections

$3\gamma$  :  $2 \times 1\gamma$  @ 511 keV and  $1 \times 1\gamma$  @ 1.157 meV;

$\rightsquigarrow$  The origin of the decay is **close** to the LOR/COR intersection.

### Partial detections

$1\gamma_{\text{COR}}^{511}$  :  $1 \times 1\gamma$  @ 511 keV;

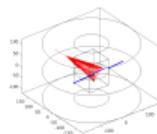
$1\gamma_{\text{COR}}^{1157}$  :  $1 \times 1\gamma$  @ 1.157 meV;

$2\gamma_{\text{LOR}}$  :  $2 \times 1\gamma$  @ 511 keV;

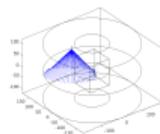
$2\gamma_{\text{COR}}$  :  $1 \times 1\gamma$  @ 511 keV and  $1 \times 1\gamma$  @ 1.157 meV.

$\rightsquigarrow$  the **restriction of the origin to LOR, COR or COR** could be **useful** too ...especially for **low-activity** imaging.

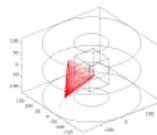
$\rightsquigarrow$  **Partial detection** classes are the **most detected** events ...



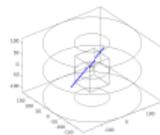
Class  $3\gamma$



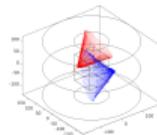
Class  $1\gamma_{\text{COR}}^{511}$



Class  $1\gamma_{\text{COR}}^{1157}$



Class  $2\gamma_{\text{LOR}}$



Class  $2\gamma_{\text{COR}}$

# Classes of detections

XEMIS2 has been developed to detect  $3\gamma$  emissions - Gallego Manzano [2016].

**But** there's more ...

## Classes of detection:

### Complete detections

$3\gamma$  :  $2 \times 1\gamma @ 511 \text{ keV}$  and  $1 \times 1\gamma @ 1.157 \text{ meV}$ ;

$\rightsquigarrow$  The origin of the decay is **close** to the **LOR/COR** intersection.

### Partial detections

$1\gamma_{\text{COR}}^{511}$  :  $1 \times 1\gamma @ 511 \text{ keV}$ ;

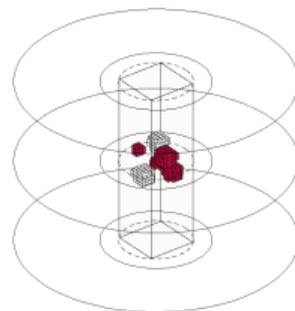
$1\gamma_{\text{COR}}^{1157}$  :  $1 \times 1\gamma @ 1.157 \text{ meV}$ ;

$2\gamma_{\text{LOR}}$  :  $2 \times 1\gamma @ 511 \text{ keV}$ ;

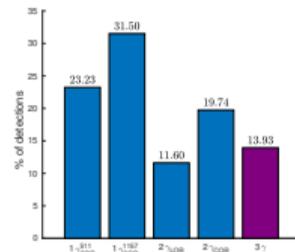
$2\gamma_{\text{COR}}$  :  $1 \times 1\gamma @ 511 \text{ keV}$  and  $1 \times 1\gamma @ 1.157 \text{ meV}$ .

$\rightsquigarrow$  the **restriction of the origin to LOR, COR or COR** could be **useful too** ...especially for **low-activity** imaging.

$\rightsquigarrow$  **Partial detection** classes are the **most detected** events ...

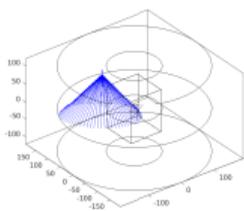


Pollux simulation:  $1e+7$   $3\gamma$  emissions in a phantom.

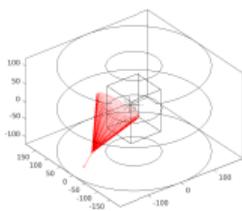


Pollux simulation: Among the  $\approx 69\%$  of emissions detected..

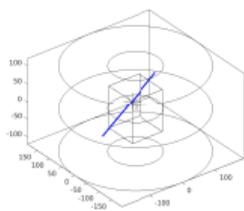
# The aim of my research



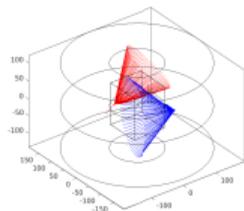
$$1\gamma_{\text{COR}}^{511} \sim 23\%$$



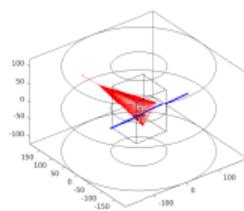
$$1\gamma_{\text{COR}}^{1157} \sim 32\%$$



$$2\gamma_{\text{LOR}} \sim 12\%$$



$$2\gamma_{\text{COR}} \sim 20\%$$



$$3\gamma \sim 14\%$$

**Our approach to solve 3γ image reconstruction:** Anything goes... partial detections too !!

Implement a **reconstruction framework** for 3γ imaging for XEMIS2 dealing with:

continuous LXe detection medium & multiple classes of detections

↔ potentially suitable for other Hybrid PET/Compton camera prototypes - Llosá and Rafecas [2023].



# Mathematics

*To model the observed phenomena*

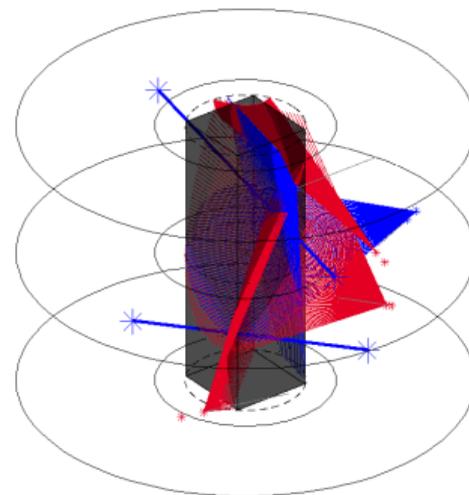
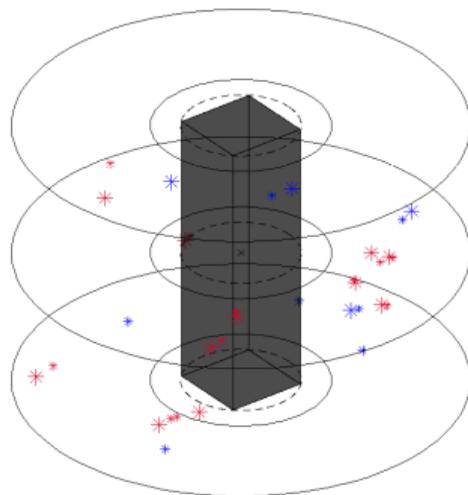
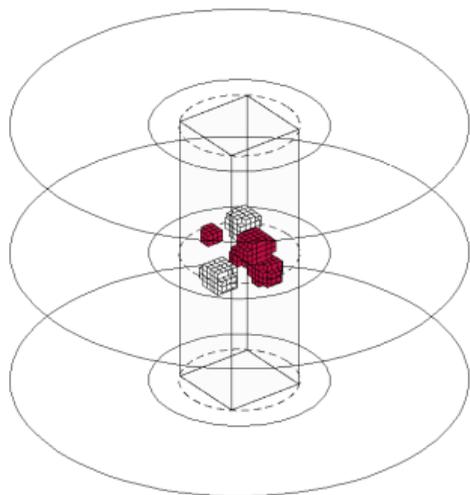
# Tomographic reconstruction

## Tomography:

From Ancient Greek:

τόμος (tomos) "slice, section" & γράφω (graphō) "to write, describe"

↪ Reconstruct the **volume of an object** from a sequence of measurements from **outside the object**.



↪ Reconstruct an **image of the radioactive distribution** in the tissue from the photons detected by **the camera**.

# Tomographic reconstruction as an inverse problem

Notations:  $J := \text{Card}(\text{voxels})$ ,  $K := \text{Card}(\text{class})$ ,  $N := \text{Card}(\text{detections})$  with  $N = \sum_k N^k$ .

## Tomographic reconstruction $\equiv$ inverse problem

Given a collection of detections:  $\mathbf{y} := \{\mathbf{y}_n\}_{n \in \llbracket 1, N \rrbracket}$ ;

Estimate the radioactive density:  $\boldsymbol{\lambda} := (\lambda_j)_{j \in \llbracket 1, J \rrbracket}$ .

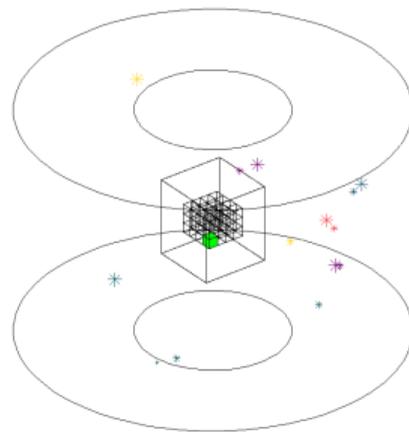
### Direct model:

To link  $\mathbf{y}$  and  $\boldsymbol{\lambda}$ :

$$\mathbf{y} := \bigcup_{k=1}^K \{\mathbf{y}_n^k\}_{n \in \llbracket 1, N^k \rrbracket} \quad \text{iid} \quad \mathbf{y}_n^k \sim \mathcal{P}(\bar{\mathbf{y}}^k)$$

where:

$$\bar{\mathbf{y}}^k := \sum_{j=1}^J T_j^k(\mathbf{y}^k) \lambda_j + \bar{\boldsymbol{\epsilon}}^k \quad \forall n \in \llbracket 1, N^k \rrbracket \quad \forall k \in \llbracket 1, K \rrbracket.$$



### Complex problem:

- $\Rightarrow$  Large dimensions of  $\mathbf{T} \xrightarrow{\bar{\boldsymbol{\epsilon}}^k=0} \mathbf{T}^{-1}$ ;
- $\Rightarrow$  perturbed measurement vector  $\mathbf{y}$ ;
- $\Rightarrow$  Small perturbation in projection space  $\rightsquigarrow$  Large error in image space.

# Expectation–Maximization algorithm

$$\hat{\lambda}^{\text{ML}} = \underset{\lambda \in \mathbb{R}_+^J}{\operatorname{argmax}} \log L(\lambda | \mathbf{y})$$

Expectation–Maximization (EM) algorithm - Dempster et al. [1977]

Determine  $\hat{\lambda}^{\text{ML}}$  for a probabilistic model depending on **hidden data**:

$\mathbf{y} := \{\mathbf{y}_n\}_{n \in \llbracket 1, N \rrbracket}$  are **known**,  $\mathbf{z} := \{\mathbf{z}_{n,j}\}_{n \in \llbracket 1, N \rrbracket, j \in \llbracket 1, J \rrbracket}$  are **hidden**  $\rightsquigarrow (\mathbf{y}, \mathbf{z})$  are the **complete data**.

$\rightsquigarrow \mathbf{z}$  is not accessible, but we can **estimate it !!**

## The EM recipe:

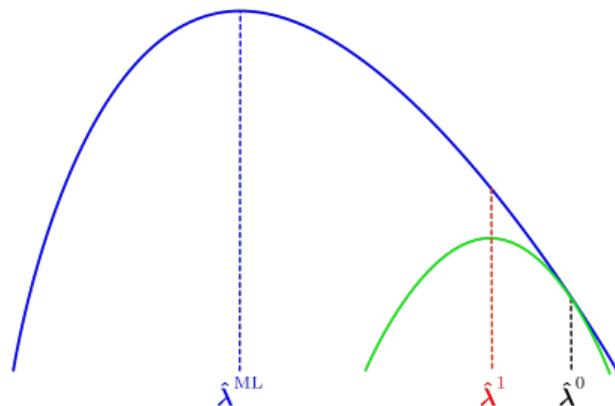
Starting from an initial guess  $\hat{\lambda}^0$  :

**E-Step:** define the auxiliary function with  $\hat{\lambda}^0$  :

$$Q(\lambda | \hat{\lambda}^0) := \mathbb{E}_{\mathbf{z} | \hat{\lambda}^0} (\log p(\mathbf{y}, \mathbf{z} | \lambda))$$

**M-Step:** Get a better estimate  $\hat{\lambda}^1$  :

$$\hat{\lambda}^1 := \underset{\lambda \in \mathbb{R}_+^J}{\operatorname{argmax}} Q(\lambda | \hat{\lambda}^0)$$



EM convergence scheme for Poisson  $\log L(\lambda | \mathbf{y})$

*Inspired by A.J.Reader*

# Expectation–Maximization algorithm

$$\hat{\lambda}^{\text{ML}} = \underset{\lambda \in \mathbb{R}_+^J}{\operatorname{argmax}} \log L(\lambda | \mathbf{y})$$

Expectation–Maximization (EM) algorithm - Dempster et al. [1977]

Determine  $\hat{\lambda}^{\text{ML}}$  for a probabilistic model depending on **hidden data**:

$\mathbf{y} := \{\mathbf{y}_n\}_{n \in \llbracket 1, N \rrbracket}$  are **known**,  $\mathbf{z} := \{\mathbf{z}_{n,j}\}_{n \in \llbracket 1, N \rrbracket, j \in \llbracket 1, J \rrbracket}$  are **hidden**  $\rightsquigarrow (\mathbf{y}, \mathbf{z})$  are the **complete data**.

$\rightsquigarrow \mathbf{z}$  is not accessible, but we can **estimate it !!**

## The EM recipe:

Starting from the new guess  $\hat{\lambda}^1$ :

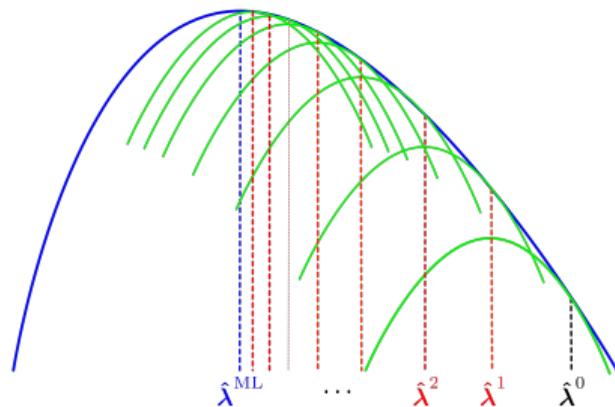
**E-Step:** define the auxiliary function with  $\hat{\lambda}^1$ :

$$Q(\lambda | \hat{\lambda}^1) := \mathbb{E}_{\mathbf{z} | \hat{\lambda}^1} (\log p(\mathbf{y}, \mathbf{z} | \lambda))$$

**M-Step:** Get a better estimate  $\hat{\lambda}^2$ :

$$\hat{\lambda}^2 := \underset{\lambda \in \mathbb{R}_+^J}{\operatorname{argmax}} Q(\lambda | \hat{\lambda}^1)$$

and repeat both steps until the convergence to  $\hat{\lambda}^{\text{ML}}$



EM convergence scheme for Poisson  $\log L(\lambda | \mathbf{y})$

Inspired by A.J.Reader

$$\hat{\lambda}^{\text{ML}} = \underset{\lambda \in \mathbb{R}_+^J}{\text{argmax}} \log L(\lambda | \mathbf{y})$$

MC-LM-MLEM - Latif et al. [2023a]

Proof of the extension of LM-MLEM proposed for PET & Compton imaging - Parra and Barrett [1998]; Wilderman et al. [1998a].

$$\left\{ \begin{array}{l} \hat{\lambda}^{(0)} = \hat{\lambda}_j^{(0)} > 0 \\ \hat{\lambda}_j^{(t+1)} := \hat{\lambda}_j^{(t)} \times \underbrace{\frac{1}{\sum_k s_j^k} \sum_{k=1}^K \sum_{n=1}^{N^k} \frac{T_j^k(\mathbf{y}_n^k)}{\sum_{j'} T_{j'}^k(\mathbf{y}_n^k) \hat{\lambda}_{j'}^{(t)} + \bar{\epsilon}_n^k}}_{\text{Correction factor}} \end{array} \right. \quad \forall j \in \llbracket 1, J \rrbracket$$

**Property:**

The algorithm converges to a  $\hat{\lambda}^{\text{ML}}$  with non-negative densities ...

Class dependant terms !!

$T_j^k(\mathbf{y}_n^k)$ : the **system "matrix"** of class  $k$  at voxel  $j$  computed **on the fly**;

$s_j^k$ : the **sensitivity** of class  $k$  at voxel  $j$ , which is **pre-computed**:

$$s_j^k := \int_{\omega \in \Omega^k} T_j^k(\omega) d\omega$$



## Computer science

*To implement efficient algorithms & simulate observed phenomena*

The sensitivity of class  $k$  which is pre-computed:

$$s^k = (s_j^k)_{j \in \llbracket 1, J \rrbracket} \quad \text{with} \quad s_j^k := \int_{\omega \in \Omega^k} T_j^k(\omega) d\omega \quad \forall k \in \llbracket 1, K \rrbracket.$$

## LOR based classes:

Let  $M$  be an emission point of an annihilation belonging to the voxel  $j$  in the FOV  $\mathcal{V}$ :

$$s_j^{\text{LOR}}(M) := \int_{\theta=0}^{\pi} \left[ \int_{\varphi=0}^{\frac{\pi}{2}} p_i \left( \frac{1}{|\cos(\varphi)|} \left( \sqrt{R_{\text{out}}^{2\gamma}(\varphi, I1)^2 - \tilde{O}P^2} - Q \right) \right) \times p_i \left( \frac{1}{|\cos(\varphi)|} \left( \sqrt{R_{\text{out}}^{2\gamma}(\varphi, J2)^2 - \tilde{O}P^2} - Q \right) \right) d\varphi \right. \\ \left. + \int_{\varphi=-\frac{\pi}{2}}^0 p_i \left( \frac{1}{|\cos(\varphi)|} \left( \sqrt{R_{\text{out}}^{2\gamma}(\varphi, I2)^2 - \tilde{O}P^2} - Q \right) \right) \times p_i \left( \frac{1}{|\cos(\varphi)|} \left( \sqrt{R_{\text{out}}^{2\gamma}(\varphi, J1)^2 - \tilde{O}P^2} - Q \right) \right) d\varphi \right] d\theta,$$

where  $\tilde{O}P := \sin(\theta)\tilde{O}M$ ,  $Q := \sqrt{R_1^2 - \tilde{O}P^2}$ .

The sensitivity of class  $k$  which is pre-computed:

$$s^k = (s_j^k)_{j \in \llbracket 1, J \rrbracket} \quad \text{with} \quad s_j^k := \int_{\omega \in \Omega^k} T_j^k(\omega) d\omega \quad \forall k \in \llbracket 1, K \rrbracket.$$

## COR based classes:

Let  $M$  be an emission point of a  $E_0$  photon belonging to the voxel  $j$  in the FOV  $\mathcal{V}$ :

$$s_j^{\text{COR}}(M) := \int_{\theta=0}^{2\pi} \left[ \int_{\varphi=0}^{\frac{\pi}{2}} \int_{v=R_{\text{in}}}^{R_{\text{out}}^{1\gamma}(\varphi, I1)} h(\varphi, \theta, v) dv d\varphi + \int_{\varphi=\frac{\pi}{2}}^{\pi} \int_{v=R_{\text{in}}}^{R_{\text{out}}^{1\gamma}(\varphi, J1)} h(\varphi, \theta, v) dv d\varphi \right. \\ \left. + \int_{\varphi=-\frac{\pi}{2}}^0 \int_{v=R_{\text{in}}}^{R_{\text{out}}^{1\gamma}(\varphi, I2)} h(\varphi, \theta, v) dv d\varphi + \int_{\varphi=-\pi}^{-\frac{\pi}{2}} \int_{v=R_{\text{in}}}^{R_{\text{out}}^{1\gamma}(\varphi, J2)} h(\varphi, \theta, v) dv d\varphi \right] d\theta,$$

with  $h(\varphi, \theta, v) := f(\varphi, \theta, v) \times C(v, \theta, \varphi)$  and

$$f(\varphi, \theta, v) := p_i \left( \frac{1}{|\cos(\varphi)|} \left( \sqrt{v^2 - \sin^2(\theta) \bar{O}M^2} - \sqrt{R_{\text{in}}^2 - \sin^2(\theta) \bar{O}M^2} \right) \right) \\ C(v, \theta, \varphi) := \int_{\beta=0}^{\pi} K(\beta | E_0) \int_{\omega=0}^{2\pi} \int_{\rho=0}^{\rho_{\text{max}}(v, \theta, \varphi, \beta, \omega)} p'_i \left( \|\vec{V}_1 \vec{V}_2\|_2 \right) d\beta d\omega d\rho$$

↪ A bit complicated to evaluate analytically, **but** useful for Pollux development.

# Sensitivity vector $s^k$ - approximation through Monte Carlo simulation

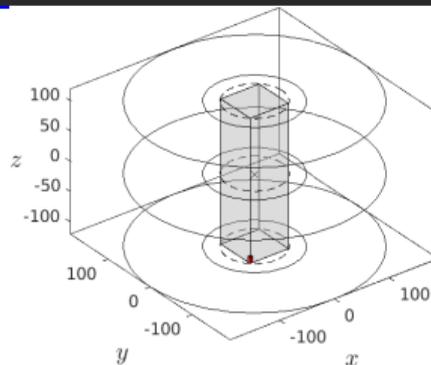
Pollux simulation to get  $\hat{s}^k$ :

FOV  $\mathcal{V}$  dimensions:  $70 \times 70 \times 240$  mm

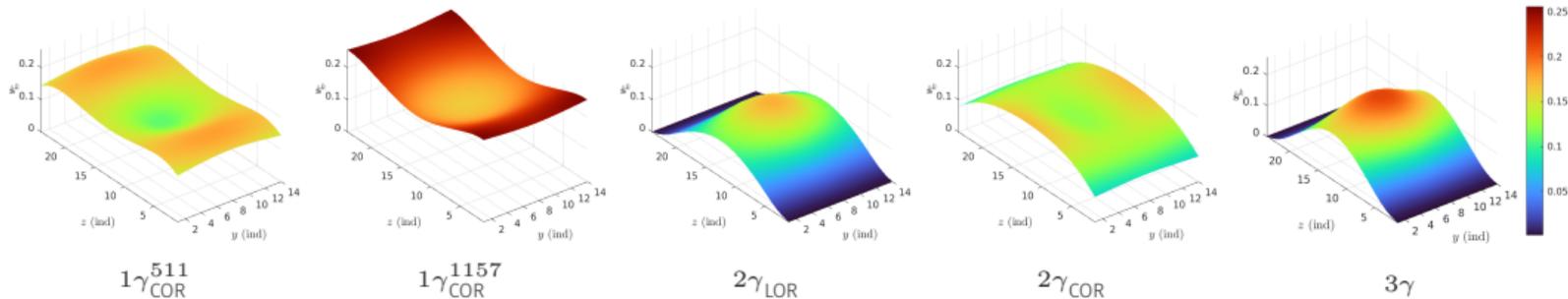
Discretization of  $\mathcal{V}$  into  $14 \times 14 \times 24$  voxels of size  $5 \times 5 \times 10$  mm;

For each voxel :  $3e+6$  uniformly generated  $3\gamma$  emissions.

↪ For each  $3\gamma$  emission, which class  $k$  is detected?



Results (after a few days)



**Visually:** the classes are **spatially complementary** e.g., in the center & at the edges of  $\mathcal{V}$ ;

**Really informative?** Multi-class Fisher information matrix - Latif et al. [2023b] ↪  $T^k(\cdot)$  is required!

# System matrix at glance

$$T_j^k(\mathbf{y}^k) := P^k(\mathbf{y}^k)A_j^k(\mathbf{y}^k) \quad \forall k \in \llbracket 1, K \rrbracket, \forall j \in \llbracket 1, J \rrbracket,$$

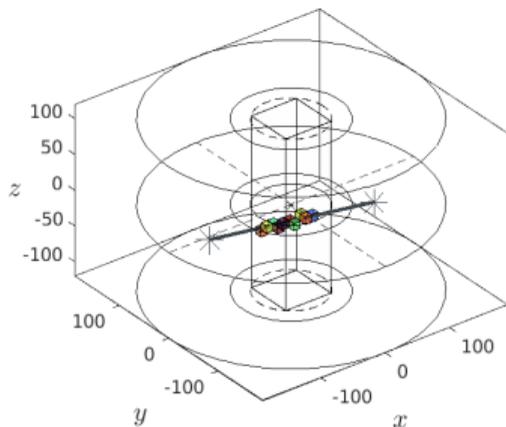
where:

$P^k(\cdot)$  relates to the **path of the photon** in the detector & **independent** of voxels  $J$ .

$A_j^k(\cdot)$  is the **projector** which links **image space** ( $J$  voxels)  $\leftrightarrow$  **data space** ( $N$  detections).

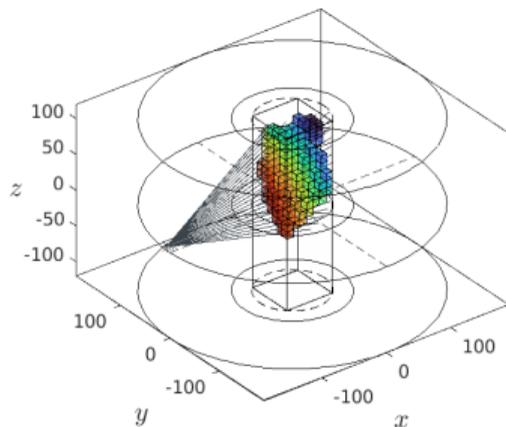
PET imaging  $\rightsquigarrow$  LOR

$$A_j^k(\cdot) \propto \text{length}(\mathcal{L} \cap v_j) - \text{Siddon [1985]}$$



Compton imaging  $\rightsquigarrow$  COR

$$A_j^k(\cdot) \propto \text{angular dist}(\mathcal{C}(V_1, \overrightarrow{V_2 V_1}, \beta), v_j)$$



$\rightsquigarrow$  Estimating  $A_j^k(\cdot)$  is **quite simple** with **PET** imaging, but a **bit harder** for **Compton** imaging.

# System matrix for Compton imaging: state-of-the-art

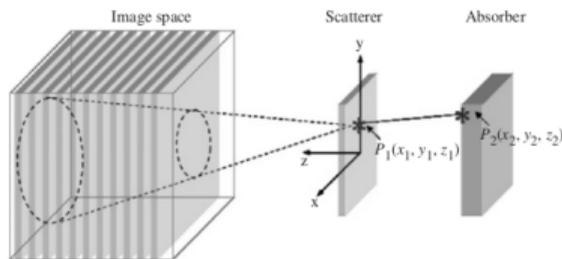
$$T_j^k(\mathbf{y}^k) \propto A_j^k(\mathbf{y}^k) := \int_{M \in v_j \cap \mathcal{C}} f_\beta(M) dv$$

↪ Surface integral on  $M \in v_j \cap \mathcal{C}$ , a **bit complicated** to evaluate analytically ...

State-of-the-art: **Cone-driven** methods for system matrix **approximation**:

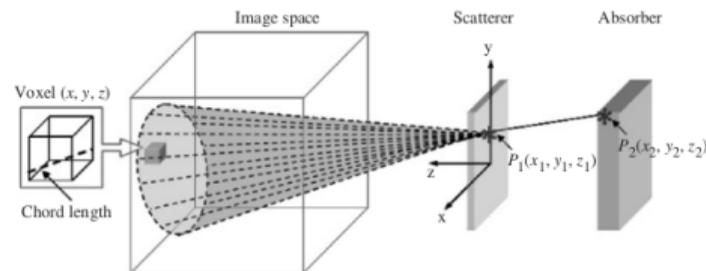
Ellipse-stacking method (ESM)

Wilderman et al. [1998b]



Ray-tracing method (RTM)

Kim et al. [2007], Siddon [1985]



• According to Kim et al. [2007], RTM  $\gg$  ESM for reconstructed image quality ... **sampling conditions ?**

**⚠ Energy uncertainties**  $\rightsquigarrow$  Shell of Response (SOR)

$$\cos \beta := 1 - \frac{m_e c^2 E_1}{E_0(E_0 - E_1)} \implies \mathcal{S}(V_1, \overrightarrow{V_2 V_1}, \tilde{\beta}, \delta\beta) := \left\{ \beta \in [\tilde{\beta} - \delta\beta; \tilde{\beta} + \delta\beta] \mid \mathcal{C}(V_1, \overrightarrow{V_2 V_1}, \beta) \right\}$$

• A **hybrid** ESM/RTM approach was proposed - Lojacono [2013], **but** the results were inconclusive ...

# System matrix for Compton imaging: our approach !

$$T_j^k(\mathbf{y}^k) \propto A_j^k(\mathbf{y}^k) := \int_{M \in v_j \cap \mathcal{S}} f_\beta(M) dv$$

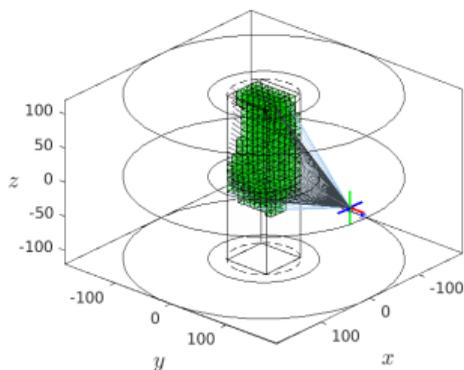
Volume integral on  $M \in v_j \cap \mathcal{S}$  due to **energy uncertainty**  $\rightsquigarrow$  a **bit complicated** to evaluate analytically ...

LS2N/CRCI<sup>2</sup>NA method: **Voxel-driven** methods for system matrix **approximation** ( $\neq$  Cone-driven)

$\rightsquigarrow$  A two-step approach using cubic voxel  $v_j \in \mathcal{V}$ :

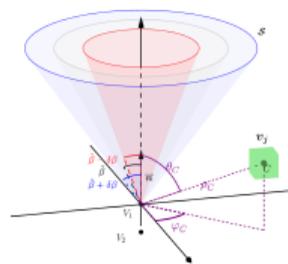
**Step 1:** Conic section screening

To find all voxels st.  $v_j \cap \mathcal{S} \neq \emptyset$

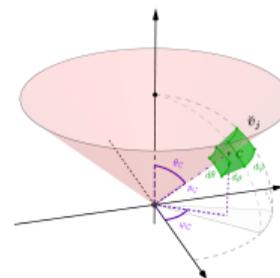


**Step 2:** Voxel deformation  $\check{v}_j$  preserving  $\text{vol}(v_j)$

To easily compute an approximation of  $\text{vol}(v_j \cap \mathcal{S})$ .



(a) Spherical coordinates



(b) Spherical box

$\rightsquigarrow$  Approximation for **non-degenerate**  $\check{v}$ :

$$\check{A}_j(\mathbf{y}) = \left( \rho_C^2 + \frac{d^2}{12} \right) \times \frac{d^2}{\rho_C \sqrt{\sin(\theta_C)}} \times I_{\check{\beta}}(\theta \mid \Theta_{\mathcal{S}}(\check{v}_j), \delta\beta)$$



What's next ?

# What's next ?

## Mathematics

- Handle pathological cases for the evaluation of  $T_j^k(\cdot)$  in Compton imaging, aka. *Galipeurs*;
- Ill-posed problem  $\oplus$  Low-activity imaging ... some *a priori* knowledge on  $\lambda$  can be helpful;  
     $\rightsquigarrow$  to implement a regularization method for the MC-LM-MLEM algorithm - De Pierro [1995]

## Informatics

- Using Pollux simulations, to implement our new methods in the CASToR framework - Merlin et al. [2018].

## Physics

- Implement energy uncertainty in Pollux simulations - Gallego Manzano [2016];
- Use more realistic detections generated by the GEANT4 framework - Agostinelli et al. [2003].

## Other

- Writing my PhD thesis manuscript ...
- a paper about *Voxel-driven method for system matrix evaluation in Compton imaging* ...

**Reconstructions with CASToR**



Thank you for your attention

Questions ?

- Agostinelli, S., Allison, J., and ... (2003). Geant4—a simulation toolkit. Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, 506(3):250–303.
- De Pierro, A. (1995). A modified expectation maximization algorithm for penalized likelihood estimation in emission tomography. IEEE Trans. Med. Imag., 14(1):132–137.
- Dempster, A. P., Laird, N. M., and Rubin, D. B. (1977). Maximum likelihood from incomplete data via the EM algorithm. 39(1):1–22.
- Gallego Manzano, L. (2016). Optimization of a single-phase liquid xenon Compton camera for 3 $\gamma$  medical imaging. Theses, Ecole des Mines de Nantes.
- Grignon, C., Barbet, J., Bardiès, M., Carlier, T., Chatal, J., Couturier, O., Cussonneau, J., Faivre, A., Ferrer, L., Girault, S., Haruyama, T., Le Ray, P., Luquin, L., Lupone, S., Métivier, V., Morteau, E., Servagent, N., and Thers, D. (2007). Nuclear medical imaging using  $\beta^+$ ,  $\gamma$  coincidences from  $^{44}\text{Sc}$  radio-nuclide with liquid xenon as detection medium. NIM-A, 571(1-2):142–145.
- Kim, S. M., Lee, J. S., Lee, M. N., Lee, J. H., Lee, C. S., Kim, C.-H., Lee, D. S., and Lee, S.-J. (2007). Two approaches to implementing projector–backprojector pairs for 3d reconstruction from compton scattered data. Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, 571(1):255–258. Proceedings of the 1st International Conference on Molecular Imaging Technology.
- Latif, M., Idier, J., Carlier, T., and Stute, S. (2023a). Multi-class maximum likelihood expectation-maximization list-mode image reconstruction. In International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine (Fully3D 2023), Stony Brook, United States.
- Latif, M., Idier, J., Carlier, T., and Stute, S. (2023b). Reconstruction multiclasse pour l'imagerie TEP 3-photons. In GRETSI'23, Grenoble, France. Groupe de Recherche et d'Etudes de Traitement du Signal et des Images.
- Llosà, G. and Rafecas, M. (2023). Hybrid PET/Compton-camera imaging: an imager for the next generation. The European Physical Journal Plus, 138(3):214.
- Lojacono, X. (2013). Image reconstruction for Compton camera with application to hadrontherapy. Theses, INSA de Lyon.
- Merlin, T., Stute, S., Benoit, D., Bert, J., Carlier, T., Comtat, C., Filipovic, M., Lamare, F., and Visvikis, D. (2018). CASToR: a generic data organization and processing code framework for multi-modal and multi-dimensional tomographic reconstruction. Phys. Med. Biol., 63(18):185005.
- Parra, L. and Barrett, H. (1998). List-mode likelihood: EM algorithm and image quality estimation demonstrated on 2-D PET. IEEE Trans. Med. Imag., 17(2):228–235.
- Siddon, R. L. (1985). Fast calculation of the exact radiological path for a three-dimensional ct array. Medical Physics, 12(2):252–255.
- Wilderman, S., Clinthorne, N., Fessler, J., and Rogers, W. (1998a). List-mode maximum likelihood reconstruction of compton scatter camera images in nuclear medicine. In 1998 IEEE NSS/MIC, pages 1716–1720.
- Wilderman, S., Rogers, W., Knoll, G., and Engdahl, J. (1998b). Fast algorithm for list mode back-projection of Compton scatter camera data. IEEE Transactions on Nuclear Science, 45(3):957–962.